Performance Modelling of Hierarchical Cellular Networks using PEPA

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Abstract. We present a performance evaluation study of hierarchical cellular networks using PEPA. These networks constitute a new application area for this process algebra formalism. We show that this formalism can easily be used to model such systems. We also show that the strong equivalence aggregation technique behind PEPA allows a significant reduction of the state space of the underlying Markov process. Using the resulting model, we derive performance criteria such as new call blocking and dropping handover probabilities.

1 Introduction

In wireless communication networks, a service area is called a cell and is defined as the area where the received signal power from a base station is stronger than the signals from all other base stations.

Depending on the expected traffic density and on their sizes, cells are of two types: large cells or macrocells and smaller cells or microcells. Usually, macrocells are deployed in rural countries and have good properties for fast users, whereas microcellular network concept has been developed to satisfy the high traffic demand in the dense urban regions and are more suitable to provide for services requiring low mobility. Hierarchical cellular systems, which combine these two network concepts, consist of two tiers of cells, a macrocellular structure overlying a microcellular one. That means that a geographical point is potentially covered by two levels of cells and a user can be assigned to one of these two levels. Generally, such a network architecture takes into account two user classes according to their speed: the pedestrians and the vehicles.

Several reuse patterns for microcellular systems have been proposed. The most known one is the system of hexagonal cells arrays in which a cluster is generally composed of 7 microcells. The square form of cells has also been investigated and several cluster sizes (1, 2, 4, 5, 8, 9, 10) have been studied. The system performance with the application of these patterns have been investigated and it has been shown that a remarkable trade-off between capacity and interference may be encountered in the 5-microcell clusters [9]. This type of clusters, a central cell surrounded by 4 peripheral cells, has also been used in [6] as a reuse pattern of the Manhattan model (Fig.1). This model, which takes its name from the city of Manhattan, consists of square blocks, representing buildings, with streets in between them. The cluster size is taken as five because this cluster size yields good performances [6]. Typically, in an urban region, this cluster shape imposes itself as the reuse pattern of the network. In our study, we will focus on hierarchical cellular networks based on this reuse pattern. The objective of

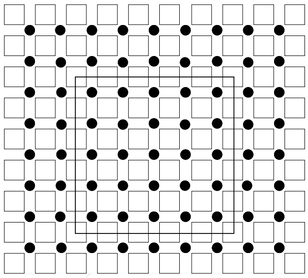

Fig. 1. The Manhattan Model

hierarchical architecture is to take advantage of the wide coverage of macrocell and the traffic capacity of microcell. However, this architecture suffers from the major drawback of microcellular systems, which is the handoff problem.

The handoff is defined as the change of radio channel used by a wireless terminal. The new radio channel can be either with the same base station (intracell handoff) or with a new base station (intercell handoff). In the case of intercell handoff, for example, where the subscriber crosses cell boundaries and moves to an adjacent cell while the call is in process, the call must be handed off to the new cell in order to provide uninterrupted service to the mobile subscriber. If the new cell does not have enough channels to support the handoff, the call is dropped. So, the handoff procedure has an important effect on the performances of the system and the probability of forced call termination must be limited because from the point of view of a mobile user, forced termination of an ongoing call is less acceptable than blocking a new call.

Various strategies were proposed in the literature in order to reduce both dropping handover and new connection blocking. The main strategies are based on one of these principles: speed-sensitive algorithm, overflowing strategies or guard channels. Speed-sensitive algorithm [3] aims to assign a user to the right cellular level according to its speed (fast user to macrocell, and slow user to microcell). Overflow channels [8] are used to take advantage of the capacity of macrocellular level when a microcell is saturated. Guard channels are used to decrease significantly the dropping handover probability by reserving channels for handover execution only. Reversible assignment capabilities [7] are also given.

In order to demonstrate their efficiency, all these proposed user managements are evaluated in terms of dropping handover and new call blocking probabilities, and traffic capacity. The performance evaluation is essentially made using simulation tools because the use of a multidimensional birth-death processes involve an exhaustive (global) state description of the network behaviour. This may then lead to models which are extremely time consuming to solve or simply impossible to solve.

Aware of this difficulty, we propose to use the process algebra formalism PEPA (Performance Evaluation Process Algebra) to study the behaviour and the performance characteristics of hierarchical cellular networks in the context of existing second generation networks such as GSM. PEPA is a compositional approach to performance modelling which has been developed by Jane Hillston [2]. Besides the fact that this formalism is very simple to use for system modelling, it allows us, as we will see it later, to have an intuitive description of our network. It provides a small but rich set of combinators which makes it powerful. Moreover, and that what makes this formalism attractive, Hillston has developed a simplification technique, based on state-to-state aggregation, for PEPA models.

In this study, we investigate two channel assignments strategies, a symmetric strategy and an asymmetric one. We also investigate the impact of the users speed on the performance of the network and the advantages to consider an overflow strategy with reversible capability rather than an overflow strategy without reversible capability.

This paper is organized as follows. Section 2 is dedicated to the description of the topology of the hierarchical cellular networks we study and the different assumptions made for the model. We then describe our PEPA model of the network and show briefly how to apply the strong equivalence aggregation technique of PEPA to simplify the model. In Section 3, we present the performance criteria we are interested in and Section 4 gives some of the obtained numerical results. Finally, we conclude with some remarks and future works in Section 5.

2 The model

In PEPA, a system is viewed as a set of *components* which carry out *activities*. Each activity is characterized by an *action type* and a duration which is exponentially distributed. Thus each activity is defined by a couple (α, r) where α is the action type and r is the *activity rate*. Because of the exponential distribution of the activity duration, Hillston has shown that the underlying Markov process of a PEPA model is a continuous time Markov process.

PEPA formalism provides a set of combinators which allows expressions to be built, defining the behaviour of components, via the activities they engage in. Below, we present informally the combinators we are interested in and which are necessary to our model. For more details about the formalism, see [2].

Constant: noted $S \stackrel{\text{def}}{=} P$, it allows us to assign names to components. To component S, we assign the behaviour of component P.

Prefix: noted $(\alpha, r).P$, this combinator implies that after the component has carried out the activity (α, r), it will behave as component P.

Choice: noted $P_1 + P_2$, this combinator represents competition between components. The system may behave either as component P_1 or as P_2. All current activities of the components are enabled. The first activity to complete distinguishes one of these components, the other is discarded.

Cooperation: noted $P_1 \underset{L}{\bowtie} P_2$, it allows the synchronization of components P_1 and P_2 over the activities in the cooperation set L. Components may proceed independently with activities whose types do not belong to this set. A particular case of the cooperation is when $L = \emptyset$. In this case, components proceed with activities independently and are noted $P_1 || P_2$.

In a cooperation, the rate of a shared activity is defined as the rate of the slowest component. The rate of an activity may be unspecified for a component and is noted $\top$. Such a component is said to be *passive* with respect to this action type and the rate of this shared activity is defined by the other component in cooperation.

2.1 Topology and Assumptions

As our study is based on hierarchical cellular networks deployed on urban regions as city centers, we assume a Manhattan model [6] of a city consisting of square-blocks, representing buildings, with streets in between them. In a such model,

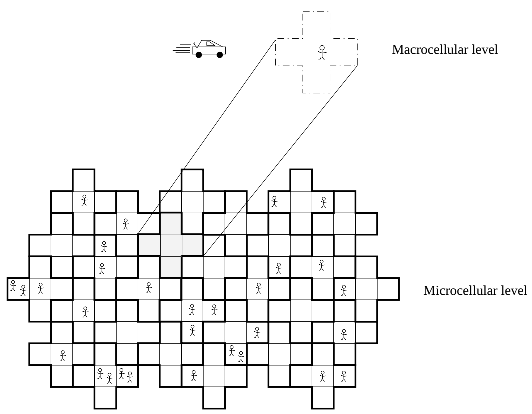

Fig. 2. Reuse pattern of Manhattan model

a reuse pattern (Fig.2) is thus composed of a macrocell overlaying a cluster of $N = 5$ microcells because of the good properties of this cluster size [9][6].

As in Manhattan model each microcell has four neighbouring cells, we consider a microcell cluster model composed of a central microcell surrounded by four peripheral cells (Fig.3). Using a FCA scheme (Fixed Channel Allocation [4]), we consider 42 channels distributed among the two layers of cells. In this study, we investigate two assignment schemes of channels between these layers: in the first one, $c_0 = 17$ channels are assigned to the macrocell and the 25 remaining channels are fairly shared between the microcells ($c_j = 5, j = 1..5$); and in the second scheme, the macrocell and all microcells have the same capacity $c_j = 7$, $j = 0..5$. Considering a homogeneous system in statistical equilibrium,

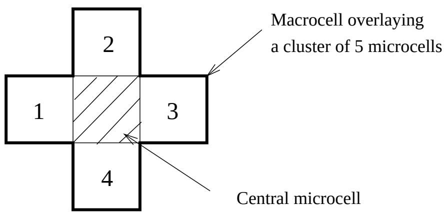

Fig. 3. The cluster model

any cluster of microcells overlayed by a macrocell has statistically the same behaviour as any other cluster of microcells overlayed by a macrocell. We use this observation to decouple a cluster from the rest of the system. That is, we can analyze the overall system by focusing on a given cluster under the condition that the neighbouring clusters exhibit their typical random behaviour independently.

Although hierarchical cellular networks are studied, we consider only one class of users. As we focus our study on dense urban regions, it seems reasonable to consider only services which require low mobility as slow vehicles and pedestrians. However, we consider two types of customers inside the cluster, the new calls and the handover requests (ongoing calls). External arrivals to the cluster consist of the handover requests coming from other clusters and the new calls initiated in that cluster. We assume that the handover requests coming from other clusters may occur only in the macrocell or the peripheral microcells. We consider that these arrivals may never occur in the central microcell.

In this study, new calls can be assigned only to the microcell level. Moreover, we consider a hierarchical cellular network using overflow strategy but without reversible capability. A request, either a new call or a handover request, initiated at the microcell level is served in its originated microcell if a channel is available. Otherwise, according to the overflow strategy, the request is overflowed to the upper level and is satisfied if a channel is free at this level. In the case where all channels are busy at both levels, the request is dropped (handover) or blocked (new call).

We assume that any geographical point of this network is covered by both microcellular and macrocellular levels, and that the whole area is crossed randomly by mobile users, according to an uniform traffic matrix.

This system is studied under usual Markovian assumptions. New call and handover request arrivals follow a Poisson process. We assume that the average new call arrival rates and the handover arrival rates are the same for all cells in the network. The session duration which represents the duration of a communication is modelled by a service time which is exponentially distributed with parameter μ. The amount of time that a user remains within a coverage cell of a given base station, called *dwell-time*, is assumed also exponentially distributed with parameter α. Similar assumptions have served telecommunications traffic engineering well for many years.

In the next section, we present the PEPA model corresponding to this system.

2.2 The PEPA model

The system as described above is modelled using six components denoted $macro$, $micro_c$ and $micro_j$, $1 \leq j \leq 4$. Component $macro$ is used to model the behaviour of the macrocell of the system. Component $micro_c$ is introduced to represent the central microcell, and $micro_j$, $1 \leq j \leq 4$, are the components which model the behaviour of the four peripheral microcells.

In the model, the external arrival process is represented by an *in* activity by the cells. The arrival rate is assumed to be λ_1 in the macrocell, λ_2 in the peripheral microcells and λ_3 in the central microcell.

As no new calls may be assigned to the macrocell, external arrivals to this cell consist only of handover requests coming from other macrocells. External arrivals to a peripheral microcell consist of handover requests coming from other clusters, and also of new calls initiated in this cell. Therefore, we set $\lambda_2 = \lambda_h + \lambda$, where λ_h is the external arrival rate of handover requests and λ the arrival rate of new calls. For the central microcell, as no arrivals from other clusters to this cell are allowed, the external arrivals to this cell are the new call arrivals. We consider that $\lambda_3 = \lambda$.

Because of the different types of cells (macro, peripheral microcell, central microcell) and the topology of the network, we make a distinction between handover calls generated by the cluster itself (Fig. 4). This distinction is based on the cell type this call is originated from and the cell type this call has to be transferred to. Thus, the arrival process of these customers is represented by a *handoff* activity indexed by the type of handover call as follows:

- activity $handoff_{up}$ represents the process which transfers a call from a microcell to the macrocell. This call is a call (a new call or a handover call) coming from outside the cluster to the microcell and because all channels in the microcell are busy, it has to be transferred to the macrocell. The rate of this activity is the external arrival rate to the microcell,
- activity $handoff_{in.c}$ represents the transfer of an ongoing call from one of the peripheral microcells to the central microcell,

- in contrast, activity $handoff_{out.c}$ represents the transfer of an ongoing call from the central microcell to one of the four peripheral microcells,
- activity $handoff_{in-up.c}$ models the process in which an ongoing call coming from a peripheral microcell and entering the central microcell, is then transferred to the macro cell because all channels of the central microcell are busy,
- activity $handoff_{out-up.c}$ models the arrival of an ongoing call from the central microcell to a peripheral microcell and because all channels of this cell are busy, the handover call is overflowed to the macro cell.

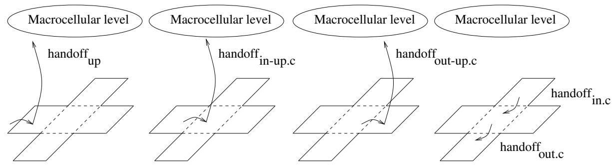

Fig. 4. The handoff activities graph

As the process behind the four last *handoff* activities is the same, the rate of these activities is also the same and it is noted α (representing the mean dwell-time in a microcell). In all cells, the service process is represented by activity *service*. As the service rate in each cell is assumed to be μ, when there are i, $1 \leq i \leq c_k$, customers in a cell, it will engage in a *service* activity at rate $i\mu$.

Now, let's give the behaviour details of the different components of the system.

Component *macro* :

When the channels of the macrocell are not all busy, if external handover calls arrive at rate λ_1, then *macro* will engage in an *in* activity at rate λ_1. If handover calls arrive from the microcells, then *macro* will engage in either a $handoff_{up}$ activity, or a $handoff_{in-up.c}$ activity, or a $handoff_{out-up.c}$ activity. In all cases, the rate of the activity is unspecified ($\top$) in component *macro* because these activities synchronize with activities of microcells generating the handover calls. The rate will be determined by the microcell which generates the handover calls.

When all channels are busy, if handover calls arrive from the microcells, even if *macro* will engage in one of the *handoff* activities, the ongoing call will be dropped and thus lost. Similarly, if external handover calls arrive when all channels are busy, even if *macro* will engage in *in* activity, the call will be blocked and lost.

If $macro_i$ describes the component behaviour when there are i customers in the macrocell, as c_0 is the channel number assigned to the macrocell, this behaviour is as follows:

$$\begin{cases} macro_0 \stackrel{\text{def}}{=} (in, \lambda_1).macro_1 + (handoff_{up}, \top).macro_1 + (handoff_{in-up.c}, \top).macro_1 \\ \qquad + (handoff_{out-up.c}, \top).macro_1 \\ \vdots \\ macro_i \stackrel{\text{def}}{=} (in, \lambda_1).macro_{i+1} + (service, i\mu).macro_{i-1} + (handoff_{up}, \top).macro_{i+1} \\ \qquad + (handoff_{in-up.c}, \top).macro_{i+1} + (handoff_{out-up.c}, \top).macro_{i+1} \\ \vdots \\ macro_{c_0} \stackrel{\text{def}}{=} (in, \lambda_1).macro_{c_0} + (service, c_0\mu).macro_{(c_0-1)} + (handoff_{up}, \top).macro_{c_0} \\ \qquad + (handoff_{out-up.c}, \top).macro_{c_0} + (handoff_{in-up.c}, \top).macro_{c_0} \end{cases}$$

Component $micro_j$: $1 \leq j \leq 4$

As for the macro component, when the channels of a peripheral microcell are not all busy, if external calls (new calls or handover calls) arrive at rate λ_2, then $micro_j$ will engage in an *in* activity at rate λ_2.

When handover requests from the central microcell arrive to a peripheral cell, if channels are not all busy, $micro_j$ will engage in a $handoff_{out.c}$ activity at an unspecified rate. If all channels are busy, $micro_j$ will then engage in $handoff_{out-up.c}$ activity. Component $micro_j$ may also engage in $handoff_{up}$ activity at rate λ_2, if external calls arrive when all channels are busy.

A peripheral microcell may generate handover calls which will ask for channels from the central microcell. If a channel is available, $micro_j$ will engage in a $handoff_{in.c}$ activity at rate $i\alpha$ where i is the number of customers in this cell. If all channels in the central microcell are busy, $micro_j$ will engage in a $handoff_{in-up.c}$ activity at the same rate.

If $micro_{ji}$ denotes the component behaviour when there are i customers in the peripheral microcell j which capacity is c_j, this behaviour is as follows:

$$\begin{cases} micro_{j0} \stackrel{\text{def}}{=} (in, \lambda_2).micro_{j1} + (handoff_{out.c}, \top).micro_{j1} \\ \vdots \\ micro_{ji} \stackrel{\text{def}}{=} (in, \lambda_2).micro_{j(i+1)} + (handoff_{out.c}, \top).micro_{j(i+1)} \\ \qquad +(service, i\mu).micro_{j(i-1)} + (handoff_{in.c}, i\alpha).micro_{j(i-1)} \\ \qquad +(handoff_{in-up.c}, i\alpha).micro_{j(i-1)} \\ \vdots \\ micro_{jc_j} \stackrel{\text{def}}{=} (service, c_j\mu).micro_{j(c_j-1)} + (handoff_{up}, \lambda_2).micro_{jc_j} \\ \qquad +(handoff_{in-up.c}, c_j\alpha).micro_{j(c_j-1)} + (handoff_{in.c}, c_j\alpha).micro_{j(c_j-1)} \\ \qquad +(handoff_{out-up.c}, \top).micro_{jc_j} \end{cases}$$

Component $micro_C$:

The $micro_C$ component differs from the other microcells only by the fact that it can receive handover calls generated by the four peripheral cells. So, it engages in the same type of activities and the same reasoning is used to describe its behaviour.

Let $micro_{Ci}$ denotes the component behaviour when there are i customers in the central microcell. As c_5 is the channel number assigned to this cell, the behaviour of this one is as follows:

$$\begin{cases} micro_{C0} \stackrel{\text{def}}{=} (in, \lambda_3).micro_{C1} + (handoff_{in.c}, \top).micro_{C1} \\ \vdots \\ micro_{Ci} \stackrel{\text{def}}{=} (in, \lambda_3).micro_{C(i+1)} + (service, i\mu).micro_{C(i-1)} \\ \qquad +(handoff_{in.c}, \top).micro_{C(i+1)} + (handoff_{out-up.c}, i\alpha).micro_{C(i-1)} \\ \qquad +(handoff_{out.c}, i\alpha).micro_{C(i-1)} \\ \vdots \\ micro_{Cc_5} \stackrel{\text{def}}{=} (service, c_5\mu).micro_{C(c_5-1)} + (handoff_{up}, \lambda_3).micro_{Cc_5} \\ \qquad +(handoff_{out.c}, c_5\alpha).micro_{C(c_5-1)} + (handoff_{in-up.c}, \top).micro_{Cc_5} \\ \qquad +(handoff_{out-up.c}, c_5\alpha).micro_{C(c_5-1)} \end{cases}$$

The system is formed by the cooperation of *macro* and the different microcells. Since the four peripheral microcells proceed independently, and cooperate with the central microcell, the system is defined as follows:

$$System \stackrel{\text{def}}{=} \Big((micro_{10} || micro_{20} || micro_{30} || micro_{40}) \underset{L}{\bowtie} micro_{C0} \Big) \underset{K}{\bowtie} macro_0$$

where L is the set of activities on which the central microcell and the peripheral microcells must synchronize. The set K contains the activities on which the macrocell and the microcells must synchronize. These two cooperation sets are defined as follows:

$L = \{handoff_{in.c},\ handoff_{out.c}\}$
$K = \{handoff_{up},\ handoff_{in-up.c},\ handoff_{out-up.c}\}$

This model, with 1 macrocell overlying 5 microcells, has 262144 states and 2895872 transitions in the case where all cells have the same capacity $c_j = 7$, $j = 0..5$. That would be extremely time consuming to solve this model. A technique to reduce the state space generated by our model is required. For that, we consider the simplification technique developed by Hillston in [2] for PEPA models. In the following, we present briefly this simplification technique and show briefly how to apply it on our model to reduce its state space.

2.3 The model simplification

The simplification technique for PEPA models is based on the state-to-state aggregation and uses the strong equivalence notion introduced by Hillston in [2]. This approach takes advantage of symmetries within a derivative set of a model, and exploits the fact that strong equivalence is a congruence.

The principle of this technique consists of applying strong equivalence aggregation as a state-to-state equivalence compositionally, replacing cooperating components by strong equivalent, lumped components [2]. Hillston showed that if strong equivalence over the derivative set of a component is used to induce a partition of the state space of the Markov process, then the corresponding aggregation will result in a Markov process. That means that the process is strongly lumpable [5].

The application of the technique does not require to construct the full derivative set of the original model and no Markov process is constructed until the aggregation procedure is completed.

In our model, the components which exhibit the same behaviour are the four peripheral microcells. Consider the component representing these microcells in the system, $micro_{10}||micro_{20}||micro_{30}||micro_{40}$. To apply the strong equivalence aggregation on this cooperating component, we proceed as follows: we consider that the component $micro_{10}||micro_{20}||micro_{30}||micro_{40}$ is composed of two components $micro_{10}||micro_{20}$ and $micro_{30}||micro_{40}$. We apply the aggregation technique on both components. We obtain two strongly equivalent, lumped components, on which we apply again the aggregation technique to obtain the final strong equivalent, lumped component, let's say M_{0000}, of the original components. Finaly, the system is modelled as follows:

$$System' \stackrel{\text{def}}{=} \left(M_{0000} \underset{L}{\bowtie} micro_{C0}\right) \underset{K}{\bowtie} macro_0$$

$System'$ is equivalent to $System$ because of the congruence property of the used equivalence.

□

All details about the aggregation technique applied on our model, the complete final model with the lumped activity sets and the behaviour of the lumped components, may be found in [1]. The model obtained has 21120 states and 181440 transitions instead of 262144 states and 2895872 transitions, in the case where all cells have the same capacity. In the second channel allocation scheme, where 17 channels are assigned to the macrocell and the remaining channels are fairly shared between the microcells, we obtain 13608 states instead of 139968 states.

These aggregation results have been obtained using, on the one hand, a minimizing Workbench developed for PEPA by Hillston's team and based on this aggregation technique, and on the other hand, a C tool we have developed and which is based on the theorem of Kemeny [5]. Both programs report the same state number, but not the same transition number. This is due to the fact that the minimizing workbench reports the number of transitions in the aggregated transition system, not the number in the Markov process. The two are different because further aggregation takes place when the rates of all the transitions between states in the Markov process are added. Moreover, in the aggregated transition system, track of multiplicity of transitions between equivalence classes is kept. This results in a greater number of transitions in the aggregated model. In the first channel assignment case, for example, we obtain 371040 transitions with the minimizing workbench of PEPA, whereas we obtain 181440 transitions with our C tool.

The Markov chain obtained using our program is solved using the Gauss-Seidel method.

2.4 The model extension

In our model, one of the assumptions we have made is the use of the overflow strategy without reversible capability in the system. This strategy implies that when a request arrives to the macrocell, if all channels of this cell are busy, this request is just lost. So when the external handover requests arrive to the macrocell from other clusters, if they are not satisfied, they can not be transferred to the microcell level even if channels may be available at that level.

In this part of the paper, we investigate the advantages to apply the overflow strategy with reversible capability to the external arrivals to the macrocell. We investigate a system in which the overflow strategy is without reversible capability for all arrival types, but for the external arrivals to the macrocell.

Because of the formalism used to model our system (PEPA), it is easy to extend our model to include other characteristics or constraints of the system. To include the new characteristic of our system, the reversible capability for the external arrivals to the macrocell, we just need to introduce a new action type $handoff_{down}$. This action type models the arrival of an ongoing call from other clusters to the macrocell and because all channels of this cell are busy, the handover call is transferred to one of the peripheral microcells. As no handover requests from other clusters to the central microcell may occur, the macrocell will not submit the handover request to the central microcell.

The new version of the model has the same components. Concerning the equations of the different components of the system, we need to add the activity $(handoff_{down}, \lambda_1)$ to the last equation of the macrocell as follows:

$$\begin{aligned} macro_{c_0} \stackrel{\text{def}}{=} & (service, c_0\mu).macro_{(c_0-1)} \\ & +(handoff_{up}, \top).macro_{c_0} + (handoff_{out-up.c}, \top).macro_{c_0} \\ & +(handoff_{in-up.c}, \top).macro_{c_0} + (handoff_{down}, \lambda_1).macro_{c_0} \end{aligned}$$

For $micro_j$, the component which models the behaviour of the peripheral microcell j, $1 \leq j \leq 4$, we need to add the activity $(handoff_{down}, \top)$ to each equation of the corresponding equations system as follows:

$$\begin{cases} micro_{j0} \stackrel{\text{def}}{=} (in, \lambda_2).micro_{j1} + (handoff_{out.c}, \top).micro_{j1} + (handoff_{down}, \top).micro_{j1} \\ \vdots \\ micro_{ji} \stackrel{\text{def}}{=} (in, \lambda_2).micro_{j(i+1)} + (handoff_{out.c}, \top).micro_{j(i+1)} \\ \qquad +(service, i\mu).micro_{j(i-1)} + (handoff_{in.c}, i\alpha).micro_{j(i-1)} \\ \qquad +(handoff_{in-up.c}, i\alpha).micro_{j(i-1)} + (handoff_{down}, \top).micro_{j(i+1)} \\ \vdots \\ micro_{jc_j} \stackrel{\text{def}}{=} (service, c_j\mu).micro_{j(c_j-1)} + (handoff_{up}, \lambda_2).micro_{jc_j} \\ \qquad +(handoff_{in-up.c}, c_j\alpha).micro_{j(c_j-1)} + (handoff_{in.c}, c_j\alpha).micro_{j(c_j-1)} \\ \qquad +(handoff_{out-up.c}, \top).micro_{jc_j} + (handoff_{down}, \top).micro_{jc_j} \end{cases}$$

The equations system describing the behaviour of the central microcell is unchanged. So, the complete system is then modelled by the following equation:

$$System \stackrel{\text{def}}{=} \Big((micro_{10} || micro_{20} || micro_{30} || micro_{40}) \underset{L}{\bowtie} micro_{C0} \Big) \underset{K}{\bowtie} macro_0$$

where

$L = \{handoff_{in.c},\ handoff_{out.c}\}$
$K = \{handoff_{up},\ handoff_{in-up.c},\ handoff_{out-up.c},\ handoff_{down}\}$

This model has the same number of states as the previous version of the model, because all states already exist. In the new version we have only additionnal transitions since we obtain 353450 transitions.

3 Performance evaluation criteria

The performance measures of the system we are interested in are the blocking probability of new calls and the dropping handover probability. The blocking probability is defined as the probability that a new fresh call is denied access to a channel, because all channels accessible by new calls at both cellular levels are busy. This probability is computed by considering the states where the new calls are lost. A new call is lost if it arrives to a full microcell and that the $handoff_{up}$ activity fails because the macrocell is also full.

The dropping handover probability, also called the forced termination probability, is the probability that a call in progress is blocked due to handoff failure during its communication. In other words, the ongoing call is blocked because there is no available channel in the cells it may be transferred to. This probability is computed by considering the states where the activities $handoff_{out-up.c}$ and $handoff_{in-up.c}$ may fail, states which correspond to a full macrocell.

As the system is assumed to have an homogeneous behaviour in the statistical equilibrium, the handover requests coming from other clusters, if not satisfied, are taken into account in the handover loss probabilities of the current cluster. Therefore, if the handover requests to the next cluster are not satisfied, they are not being counted in the handover loss probabilities of the current cluster. They are being counted in the handover loss probabilities of the next cluster.

In the following, we present some numerical results obtained for both rewards.

4 Numerical results

The analysis is conducted on the reduced model $System'$ and to compute the two performance criteria defined above it is necessary to define the input parameters. First, we consider the mean dwell time of the mobile users at both cellular levels. Both cell dwell times are adjusted proportionally to the user speed and to the cell size (Tab.1). We use a radius of $300m$ for microcells and of $900m$ for the macrocell. We consider also a standard average speed of $20km/h$. This speed is obtained using the weighted average speeds of both pedestrians and slow vehicles. Usually, for a dense urban region, 80% of the users are considered as pedestrians (3 or $4km/h$) and 20% as slow vehicles ($30km/h$). Moreover, we consider the standard session duration (μ) which is equal to $120s$.

New call origination rate	Mean dwell time (α)	
	Macrocell	Microcell
1.0 to 15.0 call per min	162 s	54 s

Table 1. Input parameters

In all our experiments, we assume the external arrival rates λ_1 and λ_h as functions of the arrival rate λ. As the arrival rate λ_h depends on the new calls initiated in neighbouring clusters, we set $\lambda_h = \gamma\lambda$. Similarly, as no new calls may be initiated in the macrocellular level and the arrival rate of handover requests to this level depends on λ, we set $\lambda_1 = \beta\lambda$. Note that $0 < \beta, \gamma < 1$.

The results obtained for dropping handover and new call blocking probabilities are depicted in Fig.5-10. All curves are plotted versus the new call arrival rate λ; we decrease the inter-arrival delay of new call requests, increasing thus the load of the system. In all these figures, we can easily see that both dropping handover and blocking probabilities increase when the load increases. Clearly, this is due to the lack of resources (channels) at both cellular levels.

1. In the first part of our experiments, we are interested in the impact of the channel allocation schemes on the blocking and dropping probabilities. The overflow strategy is without reversible capability for all arrival types. Figures 5 and 6 summarise the main results we have obtained.

- In Fig.5, we can see that, with the first channel assignment scheme, where all cells have the same capacity (7), the dropping handover probabilities obtained are higher than those obtained when we consider the second scheme. This is due to the fact that all channels of a macrocell may be requested by all microcells of the cluster. Remember, the role of the macrocell is to provide additional capacity with a pool of overflow channels to be shared between all microcells. Thus, any microcell may request for a transfer of an ongoing call to the macrocell and this request has more chances to be satisfied if an important channel number is assigned to the macrocell. In other words, even if we have less channels in each microcell (5 in the second scheme) and that may increase the handover number, the availability of a greater number of channels (17) in the macrocell will allow the macrocell to provide for a greater number of transfer requests of ongoing calls. In the case where all cells have

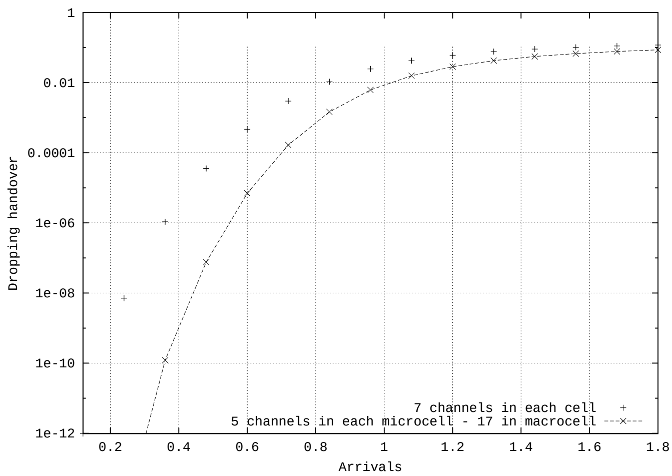

Fig. 5. Dropping handover probabilities

the same capacity, first, that will not make a big difference for the microcells, in terms of channel number, but for the macrocell, the difference will be important. That's why, in this case, we obtain higher dropping handover probabilities.
The figure shows also that for low loads, the difference between the curves is important and that difference decreases considerably for heavy loads. The reason is that when the system is very loaded, all channels of the macro will be rather inclined to be occupied and that makes not a big difference to consider one channel assignment scheme rather than the other one.

- In Fig.6, we can observe the same phenomena as in the previous figure. New call blocking probabilities obtained when considering cells with the same capacity are greater than those obtained when considering the channel assignment scheme where macrocell has an important channel number comparing to microcells. This similarity is due to the fact that new calls may be assigned only to the microcellular level. That means, when all channels of the microcell in which this call has been generated are busy, the microcell engages in a $handoff_{up}$ activity. The new call is treated as an ongoing call.

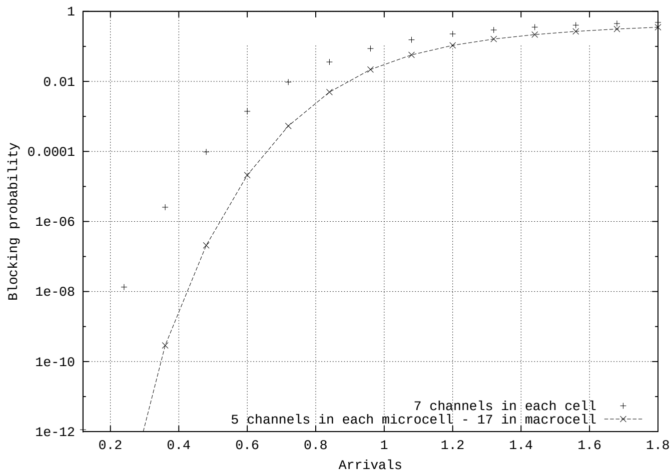

Fig. 6. New call blocking probabilities

2. The objective of the second part of the experiments is to study the impact of the user mobility (speed) on the new call blocking and dropping handover probabilities. For that, we vary the handover arrival rates to the different cells by varying the parameters β and γ. The curves corresponding to the high mobility are obtained for $\beta = 0.07$ and $\gamma = 0.21$. The other (low mobility) are obtained for $\beta = 0.05$ and $\gamma = 0.15$. Increasing the handover arrival rates implies increasing the user mobility in the system. In these experiments, we consider a system with a fixed channel assignment where all cells have the same capacity (7). We also consider the overflow startegy without reversible capability for all arrival types. Figures 7 and 8 illustrate the results obtained.

- Figure 7 shows the impact of the user mobility on the dropping handover probability. Obviously the high mobility implies greater dropping probabilities. As the user's speed increases, the number of the cells crossed by the user increases too. Therefore, the number of handovers increases. We can remark that the difference between these curves decreases as the load increases. The explaination of this phenomenon is in the fact that a slow user has a lower probability to make a handover request since he has more chance to finish his communication before arriving to the current cell borders. So he has a lower probability to see his handover request dropped. When the load is high, all channels are busy and both low and fast users will have the same probability to see their handover requests dropped.

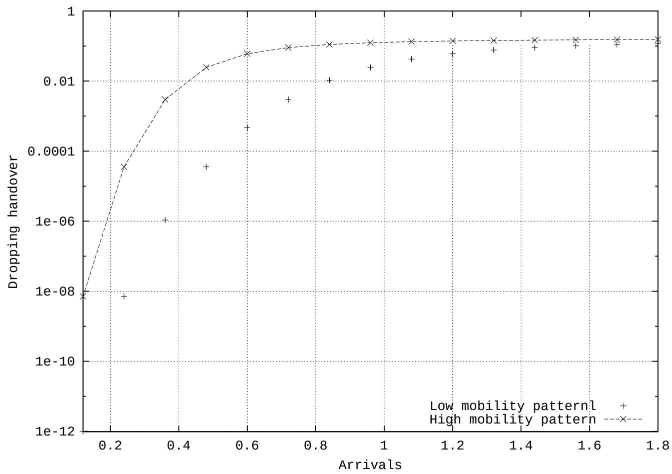

Fig. 7. Dropping handover probabilities

- Figure 8 shows the impact of the user mobility on the new call blocking probability. Similarly, we obtain lower probabilities when the user mobility is lower. In the case of new calls, we can note the same difference between the two curves. Note also that the blocking probabilities are rather greater than the dropping probabilities specially when the load increases. Remember that increasing the user mobility in the network implies increasing the handover arrival rate. So, when a new call arrives, it has more chance to be refused.

3. In the third and last part of the experiments, we study the advantages of introducing the overflow strategy with reversible capability for the external arrivals to the macrocell. In this experiment, we consider the first channel assignment scheme where all cells have the same capacity (7).

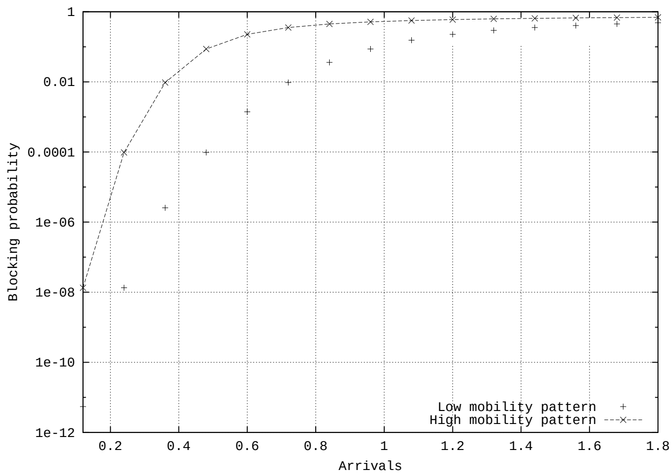

Fig. 8. New call blocking probabilities

Figures 9 and 10 summarise the results we have obtained in this experiment. In each of these figures, we present two curves, one corresponds to the system where the overflow strategy is without reversible capability and the other one corresponds to a system where the overflow strategy is with reversible capability for the external arrivals to the macrocell.

– Figure 9 shows the impact of the reversible capability on the behaviour of dropping handover probabilities. In this figure, we can see that in a system

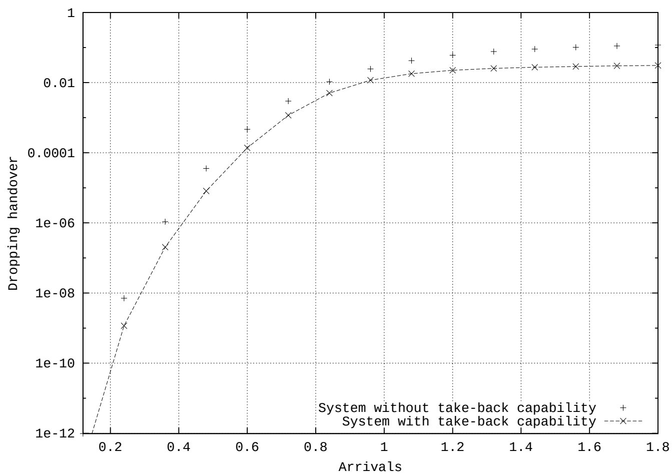

Fig. 9. Dropping handover probabilities

with reversible capability even if it only concernes the external arrivals to the macrocell, the probabilities are lower than in a system without. This may be explained by the fact that the reversible capability gives a certain flexibility to the system. So, when a handover request from another cluster arrives to the macrocell, if all channels are busy, the system gives the handover another chance to find a channel in a peripheral microcell. We can also note that the difference beween the two curves increases slightly when the load increases. This is due to the fact that when the load increases, we have more handover requests in the cluster and when the overflow strategy is without reversible capability, the losses are more important than in a system with reversible capability. However, the reversible capability will not help when the system is too loaded since at that moment all channels are busy.

– Compared to the results obtained in Figure 9, the results in Figure 10 show that the reversible capability has a very small effect on the new call blocking probabilities. Moreover, the effect even if very small is an inverse effect since the dropping handover probabilities are more important when the reversible capability is possible. This is due to the fact that the new calls do not take advantage of a system where the reversible capability is only possible for handover requests.

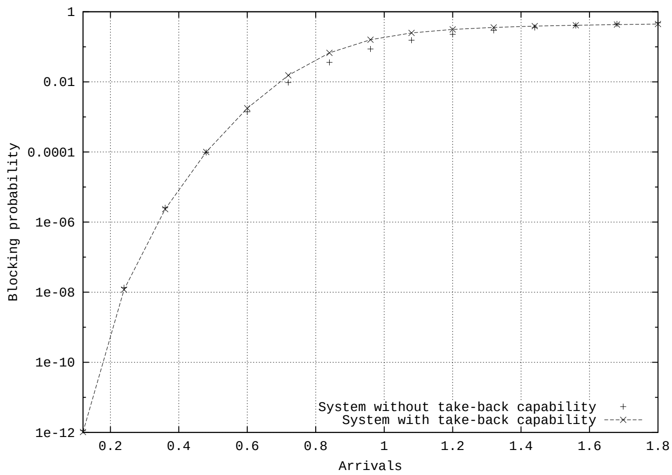

Fig. 10. New call blocking probabilities

5 Conclusion

Using PEPA language, we have described, through the definitions of components (microcells and macrocell) and activities (service, new call arrivals, handover requests), the dynamic behaviour of wireless communications within hierarchical networks. This description leads to the underlying Markov process that cannot be solved because of its large number of states. To reduce this state space, we have used a simplification technique based on the state-to-state aggregation and the strong equivalence notion. This technique reduces significantly the number of states and transitions of the model. From the model obtained, we derived performance measures of the network: dropping handover and new call blocking probabilities.

We have first explored symmetric versus asymmetric resource allocation schemes between both cellular levels, and we have shown that a channel allocation scheme assigning more resources to the macrocellular level yields better results. We then have studied the impact of the mobility criteria on both the dropping handover and blocking new call probabilities: the more the terminal speed increases, the greater the average number of handovers is. The risk to interrupt an ongoing call is then greater than a tolerate threshold. Finally, our last study compares a non-reversible strategy with a reversible one which gives the capability to hand down an ongoing call from the macrocellular level to the microcellular one. We have shown that the second policy gives better dropping handover probabilities to the detriment of blocking new call probabilities.

Hierarchical cellular networks constitute a new application domain for the process algebra formalism, PEPA. Further works will consist in the study of different hierarchical network topologies: cluster of six or seven microcells; and different user management strategies. We are also interested in the integration of two users classes within our PEPA model, because of the population characteristics traditionally met in hierarchical cellular network: fast users like vehicles and more slowly users as pedestrians. And finally, we envisage to integrate the *Markov Modulated Poisson Process (MMPP)* in PEPA functionalities to put more variance in the arrival process. This future work is motivated by the wish to use a more representative model of handover arrival process because the assumption of the exponential distribution can be not representative enough.

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